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Non-Translation Invariant Gibbs States with Coexisting Phases

I. Existence of Sharp Interface for Widom-Rowlinson Type Lattice Models in Three Dimensions*

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Abstract. We investigate the spatially inhomogeneous states of two component, $A-B$, Widom-Rowlinson type lattice systems. When the fugacity of the two components are equal and large, these systems can exist in two different homogeneous (translation invariant) pure phases one A-rich and one B-rich. We consider now the system in a box with boundaries favoring the segregation of these two phases into "top and bottom" parts of the box. Utilizing methods due to Dobrushin we prove the existence, in three or more dimensions, of a "sharp" interface for the system which persists in the limit of the size of the box going to infinity. We also give some background on rigorous results for the interface problem in Ising spin systems.

1. Introduction

This is the first part of a study of the interface between two coexisting phases. The desired goal of our study is an understanding of the interface of a continuum system; e.g., that of a liquid-vapour or a segregating binary mixture in $\mathbb{R}^3$. There are good reasons for believing that in the absence of external forces acting to spatially segregate the phases, e.g., gravity, the location of the interface would undergo very large fluctuations in the equilibrium state of the system [1, 2]. That is, if we imagine the gravitational field gradually turned off, then the mean square fluctuation of the height of the liquid-vapour interface would tend to a quantity $r(A)$, where A is the cross-sectional area of the system, with $r(A)\to\infty$ as $A\to\infty$ [presumably $r(A)\sim\ln A$[1]]. It is expected nevertheless that it is possible to define

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1 This is in analogy with the behavior of coupled harmonic oscillators (Gaussian spins) in two dimensions

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