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# Non-Markovian Quantum Stochastic Processes and Their Entropy

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**Abstract.** A definition of a quantum stochastic process (QSP) in discrete time capable of describing non-Markovian effects is introduced. The formalism is based directly on the physically relevant correlation functions. The notion of complete positivity is used as the main mathematical tool. Two different but equivalent canonical representations of a QSP in terms of completely positive maps are derived. A quantum generalization of the Kolmogorov-Sinai entropy is proved to exist.

## 1. Introduction

The purpose of this paper is to introduce the notion of a quantum stochastic process (QSP) and to define the entropy of a stationary QSP with a normal invariant state. Our formalism is intended to describe the irreversible time evolution of a finite open quantum system in contact with arbitrary reservoirs and measuring instruments. This time evolution is not assumed to be Markovian, i.e. the system has a memory.

Our definition of a QSP differs from that of E. B. Davies, which includes a Markov condition [1]. In fact most previous work on the dynamics of open quantum systems have treated the Markovian case only. Examples of recent work on Markovian systems are Davies' rigorous results on Markovian master equations [2], the theory of dynamical semigroups [3], and the derivation of the general form of their generators [4–6].

The abstract projection method introduced by Nakajima and Zwanzig, however, leads to generalized master equations (GME) which are non-Markovian in general [7–9]. The exact solution of the GME is equivalent to the complete description of the dynamics of the total system including reservoirs. Approximate treatments suffer from the defect that conditions sufficient to ensure the positivity preserving property of the time evolution are not known. Consequently it is difficult to stop negative probabilities from cropping up [10]. The alternative approach of generalizing the Langevin method to the non-commutative case also runs into severe difficulties [11].

In the commutative case the theory of stochastic processes gives a sound basis for discussions of similar fundamental problems. Therefore it seems worthwhile to

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