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Convergence to Equilibrium in a System of Reacting Polymers*

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Abstract. Chemical kinetics of a system of reacting polymers is modelled by an equation which shares certain properties with Boltzmann's equation. Being more tractable, however, this evolution may be of an illustrative value for the latter. The existence and uniqueness of solutions are analysed. We derive an entropy production inequality which is used to prove global exponential decay of the free energy. With its aid a uniform rate for strong convergence to equilibrium is proven. The generators of the linearized flow at the vicinity of the equilibria are diagonalized.

I. Introduction

Many substances form long-chain polymers of varying length. The distribution of the length of the polymers is determined by the dynamical equilibrium between competing reactions; that of degradation, caused by the breaking of bonds, and recombination in which two linear polymers join at their ends.

In a simple model of such a system the density function (whose argument is the length of a polymer) obeys a dynamical equation which shares certain properties with Boltzmann's equation. However, as it turns out, this equation is more amenable to analysis and as such it may be of illustrative value. In particular, it offers an example in which an analog of the H-theorem can be used directly, with the aid of a new inequality, to prove a global convergence to equilibrium.

The following notation is being used:

$\hat{c}(t, n)$ is the number of polymers of n units; A is a quantity of the order of magnitude of the total number of polymers (e.g. Avogadro number); δ is the length of a building unit of the polymers; $x = n \cdot \delta$ is the length of a polymer; $c(x) = \hat{c}(x/\delta)\delta^{-1}A^{-1}$, i.e. $c(x)dx$ is the number, in the units of A, of polymers whose length is in $[x, x+dx)$.

Assuming that all the bonds (of which there are $n-1$ in a polymer of length n) break independently and with the same rate and that the probability for two

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