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On the Proof of the Positive Mass Conjecture in General Relativity

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Abstract. Let M be a space-time whose local mass density is non-negative everywhere. Then we prove that the total mass of M as viewed from spatial infinity (the ADM mass) must be positive unless M is the flat Minkowski space-time. (So far we are making the reasonable assumption of the existence of a maximal spacelike hypersurface. We will treat this topic separately.) We can generalize our result to admit wormholes in the initial-data set. In fact, we show that the total mass associated with each asymptotic regime is non-negative with equality only if the space-time is flat.

0. Introduction

This is the second part of our paper on scalar curvature of a three-dimensional manifold and its relation to general relativity. The problem in general relativity that we address is the following: An isolated gravitating system having non-negative local mass density must have non-negative total mass, measured gravitationally at spatial infinity.

Mathematically, the positive mass conjecture can be described as follows: Let N be a three dimensional Riemannian manifold with metric tensor g_{ij}. Then an initial set consists of N and a symmetric tensor field h_{ij} so that $\mu \geqq \left|\sum_a J^a J_a\right|^{1/2}$ where μ and J are defined by

$$\mu = \tfrac{1}{2}\left(R - \sum_{a,b} h^{ab}h_{ab} + \left(\sum_a h^a_a\right)^2\right),$$

$$J^a = \nabla_b\left[h^{ab} - \left(\sum_c h^c_c\right) g^{ab}\right]$$

where R is the scalar curvature of our metric.

If N is a spacelike hypersurface in a space time so that g_{ij} is the induced metric and h_{ij} is the second fundamental form, then the above condition says that the apparent energy-momentum of the matter be timelike.

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