Commun. math. Phys. 64, 233—268 (1979)

Communications in
Mathematical
Physics

Instantons and Fermions in the Field of Instanton

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Abstract. The number of instantons and the number of zero fermion modes in the field of instanton are calculated. The quantum fluctuations of instantons are studied.

Section 1. Introduction

Let us consider gauge fields taking values in the Lie algebra of simple compact non-abelian Lie group G. The topological number of the field A_α having finite euclidean Yang-Mills action $S=\frac{1}{2g^2}\int\langle F_{\alpha\beta},F^{\alpha\beta}\rangle dV$ can be defined as

$$q=\frac{1}{16\pi^2}\int\langle F_{\alpha\beta},*F^{\alpha\beta}\rangle dV\,. \tag{1}$$

It is proved in [1] that $S\geqq 8\pi^2g^{-2}|g|$ and $S=8\pi^2g^{-2}|g|$ if

$$F_{\alpha\beta}=*F_{\alpha\beta} \tag{2}$$

for $q\geqq 0$

$$F_{\alpha\beta}=-*F_{\alpha\beta} \tag{3}$$

for $q\leqq 0$. Here $F_{\alpha\beta}$ denotes the strength of the field A_α and $*F_{\alpha\beta}$ the dual tensor. Following [2] we use the name instanton for solutions of the duality Eq. (2) and the name anti-instanton for solutions of (3). The Eqs. (2), (3) are conformally invariant and therefore we can replace the fields on the euclidean space by the fields on the sphere S^4 in (2), (3). The instantons having topological number 1 were found in [1] and used in many papers to understand the structure of quantum gauge theories. The quantum fluctuations of such instantons were studied in [2–4]. The examples of instantons having arbitrary topological number were given in [5–7]. G. 't Hooft found a $5q$-parameter family of instantons having topological

0010-3616/79/0064/0233/$07.20