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The Dynamical Degrees of Freedom in Spatially Homogeneous Cosmology*

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Abstract. The true analogues of superspace and conformal superspace for spatially homogeneous cosmology are introduced and discussed in relation to the kinematics of the evolution of Cauchy data from a spatially homogeneous initial value surface using a spatially homogeneous lapse function. Having fixed the slicing of spatially homogeneous spacetimes to be the natural one, an obvious restriction on the freedom of choice of the shift vector field occurs, and its relation to the three-dimensional diffeomorphism gauge group of the problem is explained. In this context the minimal distortion shift equation of Smarr and York naturally arises. Finally these ideas are used to simplify the dynamics.

I. Preliminaries

We first introduce a good deal of notation and standard results concerning a Lie group and its associated Lie algebras and groups[1]. Let G be a simply connected n-dimensional Lie group ($n=3$ will be our principal application) with identity element u_0, $\mathscr{D}(G)$ its diffeomorphism group, $\mathfrak{X}(G)$ the Lie algebra of vector fields on G and $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$ the respectively left invariant and right invariant n-dimensional Lie subalgebras of $\mathfrak{X}(G)$.

Let $\mathfrak{X}(G)^*$ be the space of 1-forms on G and $\mathfrak{g}^*$ and $\tilde{\mathfrak{g}}^*$ the respectively left invariant and right invariant n-dimensional subspaces of $\mathfrak{X}(G)^*$. These may be identified with the dual spaces of $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$, and the entire space of left or right invariant tensor fields on G may be identified with the tensor algebra over $\mathfrak{g}$ or $\tilde{\mathfrak{g}}$ respectively. $\mathfrak{g}$ is usually called the Lie algebra of G. A choice of basis $e=\{e_a\}$ of $\mathfrak{g}$ determines a basis $\tilde{e}=\{\tilde{e}_a\}$ of $\tilde{\mathfrak{g}}$ (uniquely defined so that e and $\tilde{e}$ coincide at the identity) and dual bases $\{\omega^a\}$ and $\{\tilde{\omega}^a\}$ of the corresponding dual spaces. Both e and $\tilde{e}$ are global frames on G with respective dual frames $\{\omega^a\}$ and $\{\tilde{\omega}^a\}$.

The group of automorphisms of G

$$\mathrm{Aut}(G)=\{h\in\mathscr{D}(G)\,|\,h(u_1u_2)=h(u_1)h(u_2);\ \forall u_1,u_2\in G\}$$

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