

# Frobenius Theory for Positive Maps of von Neumann Algebras

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**Abstract.** Frobenius theory about the cyclic structure of eigenvalues of irreducible non negative matrices is extended to the case of positive linear maps of von Neumann algebras. Semigroups of such maps and ergodic properties are also considered.

## 1. Introduction

The spectral theory of positive maps has its origin in the classical work of Perron [1] and Frobenius [2], who considered the case of matrices with positive entries on finite dimensional vector spaces. For a compact exposition of Perron-Frobenius results see [3]. Let us distinguish two types of results in this theory. The first, due to Perron [1], is concerned with the existence and uniqueness of the maximal eigenvalue, the second, due to Frobenius [2], is concerned with the cyclic structure of the spectrum. Frobenius showed more particularly that a non negative irreducible matrix has always a simple eigenvalue  $r$  such that all other eigenvalues are contained in a circle of radius  $r$  around the origin. If the matrix is normalized such that  $r = 1$  then the eigenvalues on the unit circle form a finite subgroup of the circle group which maps the system of all eigenvalues into itself.

In this paper we extend Frobenius results to the case of 2-positive maps of von Neumann algebras. Let us first give some references to previous work. As the literature is quite extensive, especially concerning extensions of Perron's results, we shall mainly mention work related to Frobenius results (for additional references see [4]).

Frobenius type of results for compact operators on commutative  $C^*$ -algebras and ordered vector spaces can be found in Krein and Rutman [5], who also extended Jentsch's work [6] on Perron type of results. For other extensions in the case of ordered vector spaces see e.g. [7]–[9].

Automorphisms of commutative  $C^*$ -algebras have been studied particularly in connection with ergodic theory, originating from classical work by Koopman [10], Carleman [11] and von Neumann [12], see [13]. Results of Frobenius type for groups of automorphisms in the general case of non commutative  $C^*$ -algebras have been obtained by Størmer [14].