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Dissipations on von Neumann Algebras

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Abstract. We extend a characterisation by Lindblad of complete normal dissipations on hyperfinite von Neumann algebras to general semifinite von Neumann algebras.

Introduction

The time-development of certain quantum systems can be represented by one-parameter semigroups of completely positive maps on the associated C^*-algebras (see [4] for a discussion of the physical justification for this). When the semigroup is norm-continuous the infinitesimal generator is a bounded linear map on the C^*-algebra, and Lindblad [4] gives a characterisation of those linear maps which are infinitesimal generators of such semigroups. These he calls *complete dissipations*.

If we now take a von Neumann algebra $\mathscr{A}$ and look at complete normal dissipations on $\mathscr{A}$, we would like to prove a result corresponding to the theorem that every derivation on a von Neumann algebra is inner. In [4], Lindblad shows that if $\theta:\mathscr{A}\to\mathscr{A}$ is completely positive then $\gamma_\theta:\mathscr{A}\to\mathscr{A}$ defined by

$$\gamma_\theta(a)=\theta(a)-\tfrac{1}{2}\{\theta(1)a+a\theta(1)\} \tag{1}$$

is a complete dissipation on $\mathscr{A}$, and it is clear that γ_θ is normal if and only if θ is.

Definition. A complete dissipation γ on a C^*-algebra $\mathscr{A}$ is called *inner* if $\gamma-\gamma_\theta$ is an inner derivation for some completely positive map θ on $\mathscr{A}$.

Lindblad shows in [4] that every complete normal dissipation on a hyperfinite von Neumann algebra $\mathscr{A}$ is inner. In [5] he uses the general theory of cohomology of operator algebras to show that the same is true for any type I von Neumann algebra, except that in this case he can only show that the range of the completely positive map θ is contained in $\mathscr{B}(H)$, where $\mathscr{A}$ is considered as a weak-operator closed subalgebra of $\mathscr{B}(H)$ containing the identity map. However, since any type I von Neumann algebra is injective, there is an expectation from $\mathscr{B}(H)$ onto $\mathscr{A}$, so by the remark at the end of [5] we can choose θ with range contained in $\mathscr{A}$.

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