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Asymptotic Completeness for Quantum Mechanical Potential Scattering*

I. Short Range Potentials

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Abstract. A new (geometrical) proof is given for the asymptotic completeness of the wave operators and the absence of a singular continuous spectrum of the Hamiltonian for potentials which decrease faster than in the Coulomb case, the space dimension is arbitrary.

Introduction and Results

Quantum mechanical potential scattering is completely under control for most potentials of interest. As long as the potential vanishes at infinity fast enough to exclude the Coulomb potential the completeness of the ordinary wave operators can be proved using eigenfunction expansions (see e.g. [1] and references given therein) or other methods in special cases.

Instead of using these rather abstract methods we give a new proof for the completeness of the wave operators and the absence of a singular continuous spectrum in the Hamiltonian which follows the intuition of how a scattering particle behaves in space and time. That this "geometrical" approach to the completeness problem is the natural one was pointed out to me years ago by R. Haag. This point of view has also recently been advocated by Deift and Simon [8], Simon [7].

The main idea of our proof is as follows: a state from the continuous spectral subspace of the Hamiltonian is known [6] to leave in the time-mean any finite region of space. Such a far out localized state is decomposed into its outgoing components where $\mathbf{x} \cdot \mathbf{p}$ is positive (up to a tail) and the remaining incoming components. The outgoing components cannot interact any more, i.e. $(\Omega_- - \mathbb{1})$ is small on them, thus they lie in the range of the outgoing wave operator Ω_-. Similarly the incoming components cannot have interacted in the past. This is used to

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