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Characteristic Exponents and Strange Attractors

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Abstract. Iterates of maps in the family $f(x,y)=(y+1-Ax^2, Bx)$ (see [1]) are investigated. Characteristic exponents $C_p = \lim_{n\to\infty} (1/n)\log\|df^n(p)\|$ are estimated numerically. Further numerical investigations indicate that finite $C_p > 0$ corresponds to a strange attractor. When C values are calculated for B fixed and A in an interval, one finds dispersed among $C>0$ values many small subintervals for which $0>C$. On each such subinterval there appear to be attractors of period $k, 2k, 4k, \ldots$ the period doubling as A increases. Many different values of k have been observed. A theorem is proved for $A>0$, $1>B>0$ describing an explicit compact set K (depending on A and B) such that all non-divergent asymptotic behavior takes place in K.

1. Introduction

Let f be a mapping of R^m to R^m defining a "time evolution" $p_n = f(p_{n-1})$. By sensitive dependence on initial condition we mean that for q_0 near p_0 the distance $\|p_n - q_n\|$ is growing rapidly as n increases (cf. Ruelle [4, 5]).

If f is differentiable, we can define this concept as follows: denote $f^n = f\circ \ldots \circ f$, i.e. the n-th iterate of f and let $df^n(p)$ be the derivative of f^n at p, i.e. the $m\times m$ matrix $(\partial f_i^n/\partial x_j)$. Then f has sensitive dependence on initial condition for a set U if for each $p\in U$

1) the set $\{f^n(p)\ n=0,1,\ldots\}$ is bounded
2) $\lim_{n\to\infty} (1/n)\log\|df^n(p)\| = C_p > 0$.

In this paper we present the results of numerical experiments performed on a particular family of diffeomorphisms mapping R^2 to R^2. This family was discussed

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