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Fluid Dynamical Limit of the Nonlinear Boltzmann Equation to the Level of the Compressible Euler Equation

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Abstract. The nonlinear Boltzmann equation for a rarefied gas is investigated in the fluid dynamical limit to the level of compressible Euler equation locally in time, as the mean free path ε tends to zero. The nonlinear hyperbolic conservation laws obtained as the limit are also the first approximation of the Chapman-Enskog expansion.

§ 1. Introduction

The dimensionless Boltzmann equation in the kinetic theory of gases can be written for the mass density distribution function $F(t, x, v)$, $t \geqq 0$, $x \in \mathbb{R}^3$, $v \in \mathbb{R}^3$ in the form (cf. [4])

$$\frac{\partial F}{\partial t} + \Sigma v_j \frac{\partial F}{\partial x_j} = \frac{1}{\varepsilon} Q(F, F), \tag{1.1}$$

where ε is the mean free path and

$$Q(F, G) = \tfrac{1}{2} \int (F'G'_* + F'_*G' - FG_* - F_*G) V r dr d\phi dv_* . \tag{1.2}$$

Here, $V = |v - v_*|$, v' and v'_* are the velocities after the interaction of the molecules whose velocities were v, v_* before the interaction, and r, ϕ are the polar coordinate in the impact plane. Also $F_* = F(t, x, v_*)$, $F' = F(t, x, v')$, $F'_* = F(t, x, v'_*)$ and G_*, G', G'_* are defined analogously. Define the summational invariants

$$\{\Psi_j\}_{j=1}^5 \equiv \{1, v_j (j = 1, 2, 3), v^2\}, \tag{1.3}$$

which satisfy

$$\int \Psi_j Q(F, G) dv = 0 \quad \text{for} \quad j = 1, 2, \ldots, 5 . \tag{1.4}$$

The hydrodynamical quantities are defined as follows: The mass density and fluid flow velocity are given by

$$\varrho(t, x) \equiv \int F(t, x, v) dv , \tag{1.5}$$

$$u(t, x) \equiv \frac{1}{\varrho} \int v F(t, x, v) dv . \tag{1.6}$$

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