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N Body Scattering in the Two-Cluster Region*

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Abstract. We extend Combes' result on completeness of N-body scattering at energies below the lowest 3-body threshold from potentials with $|x|^{-\nu-\varepsilon}$ falloff ($\nu \equiv$ number of dimensions for each particle) to central potentials with $|x|^{-1-\varepsilon}$. We also treat the scattering of electrons from neutral atoms in the two cluster region.

§1. Introduction

This paper is a contribution to our program [2, 8, 3] of using geometric ($\equiv$ configuration space) methods to study multiparticle non-relativistic quantum mechanical systems. Indeed, it can be viewed as an addendum to Section 3 of [8].

Consider the Hamiltonian of N particles in ν-dimensions, i.e.,

$$\tilde{H} = -\sum_{i=1}^{N} (2m_i)^{-1} \Delta_i + \sum_{i<j} V_{ij}(\vec{r}_i - \vec{r}_j)$$

on $L^2(R^{N\nu})$. As is traditional, we separate out the center of mass and consider the "reduced" operator, H on $L^2(R^{(N-1)\nu})$. Corresponding to any decomposition α of $\{1, \ldots, n\}$ into two non-trivial subsets $C_1^{(\alpha)}, C_2^{(\alpha)}$, H decomposes into $H(C_1^{(\alpha)}) + H(C_2^{(\alpha)}) + T_\alpha + I_\alpha$ where I_α is the interaction between clusters, i.e.,

$$\sum_{i \in C_1^{(\alpha)},\, j \in C_2^{(\alpha)}} V_{ij}$$

and T_α is the relative kinetic energy of the clusters.

One defines

$$\Sigma_{2,\alpha} = \inf(\sigma(H(C_1^{(\alpha)}) + H(C_2^{(\alpha)})))$$

and

$$\Sigma_{3,\alpha} = \inf(\sigma_{\text{ess}}(H(C_1^{(\alpha)}) + H(C_2^{(\alpha)})))$$

where the operator is viewed as acting on $L^2(R^{(N-2)\nu})$. Finally, $\Sigma_2 = \min(\Sigma_{2,\alpha})$,

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