Commun. math. Phys. 58, 19—29 (1978)

Communications in Mathematical Physics

Correlation Inequalities and Uniqueness of the Equilibrium State for the Plane Rotator Ferromagnetic Model

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Abstract. We derive new inequalities for the plane rotator ferromagnetic model and use them to obtain the following results:

1) If the model is isotropic, the derivability of the free energy as function of the magnetic field h implies the existence of a unique translation invariant Gibbs state and if furthermore $h=0$ all Gibbs states are invariant by rotation of the spins.

2) If the model is anisotropic the above assertion holds for h non-zero.

3) If the model is anisotropic then there are at most two extremal translation invariant Gibbs states for almost all values of the anisotropy parameter.

1. Introduction

In a recent work [1] Lebowitz derived new inequalities for the ferromagnetic Ising model which are very useful to obtain information about the number of pure phases that can coexist in such systems. In the following section we derive the analogous inequalities for the plane rotator ferromagnetic model and we also observe that the inequalities of Ginibre [2] can be slightly generalized. Using the fact that the translation invariant Gibbs states can be defined in terms of tangent functionals to the free energy (see [16, 3, 4]), we prove in Section 3 the results mentioned in the abstract. Among them the unicity of the translation invariant Gibbs state for the isotropic model at zero magnetic field was obtained by Bricmont et al. [5] quite recently. We refer the reader to the Section 4 for further comments.

2. Inequalities

Let $\Lambda=(1,\dots,n)$ be a finite set of sites which we shall think as a subset of a regular d-dimensional lattice $\mathscr{L}$, say $\Lambda\subset\mathscr{L}=\boldsymbol{Z}^d$. The spin at the site $i\in\Lambda$ is described by a two-dimensional unitary vector

$$\sigma_i=(x_i,y_i)=(\cos\theta_i,\sin\theta_i);\qquad \theta_i\in[0,2\pi].$$

0010-3616/78/0058/0019/$02.20