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Power Spectra of Nonlinearly Coupled Waves

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Abstract. Swinney et al. (1977) have found that turbulence in rotational Couette flow results from a small number of instabilities. They have raised the question of whether these instabilities conform with the ideas of Ruelle and Takens (1971). We show that a simple model of the Couette flow yields power spectra similar to those seen in the experiments. The model is consistent with the Ruelle and Takens picture.

1. Introduction

Recent experiments on circular Couette flow by Gollub and Swinney [1] and Swinney, Fenstermacher, and Gollub (SFG) [2] have revealed a series of transitions in the power spectrum of the radial component of velocity at a point midway between the two cylinders. At low Reynolds numbers, these investigators find spectra consisting of spikes. The spectra correspond to periodic or quasiperiodic motions of the fluid. At a well defined Reynolds number, the spikes disappear leaving a series of broad humps in the power spectrum. (Only one hump was visible in the earlier experiment.) It has been asked whether these broad power spectra might be produced by a "strange-attractor" [3, 4] solution of the governing equation. We have constructed a simple model of nonlinearly coupled waves which has stochastic solutions when three or more waves have sufficiently positive growth rates. The power spectra of these systems is qualitatively similar to those observed by SFG.

2. The Model

A realistic theoretical study of the Couette flow transitions observed by SFG would require solution of the three-dimensional Navier Stokes equation. However, it is of some interest to investigate whether nonlinear coupling between a small number of waves could yield broad power spectra similar to those observed by SFG. Photographs of the flows studied by SFG reveal phenomena similar to those observed by Coles [5]. At low rotations speeds, the flow is azimuthal. As the Reynolds number is increased, Taylor cells appear in the flow. The next instability

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