Commun. math. Phys. 57, 165—171 (1977)

Communications in Mathematical Physics

Correlation Inequalities and Equilibrium States. II*

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Abstract. The hierarchy of some relevant correlation inequalities is settled, together with their relation to equilibrium conditions.

I. Introduction

Recently it was proved that the Roepstorff upper bound [1] for what we call the Duhamel two-point function, defined for a time invariant state, is equivalent with the KMS-condition for equilibrium states [2]. Along the same lines Sewell [3] used an other inequality for correlation functions and proved that it yields the KMS-condition. This technique showed to be a powerful method to prove the equivalence of KMS with local variation principles.

Here we give a proof of the equivalence of the two inequalities for time invariant states (Theorem II.4). Therefore for practical services the one or the other may be used. The Roepstorff upper bound seems to be more directly connected to physical quantities [4] whereas the other inequality looks interesting as it is linear in the Hamiltonian.

Furthermore recently a bunch of correlation inequalities has been derived for KMS-states [1–6]. Theorem II.4 yields now more elegant and more direct proofs of these inequalities. In this note we consider more five inequalities. It turns out that in general they are not equivalent to KMS or to the other conditions in Theorem II.4. Therefore the question of hierarchy between these inequalities is raised and solved.

II. Equilibrium Conditions

Let $\mathcal{M}$ be a Neumann algebra on a Hilbert space $\mathcal{H}$. Let H be a self-adjoint operator on $\mathcal{H}$ and $U_t = \exp it H$, $t \in \mathbb{R}$ such that $t \to U_t \cdot U_t^*$ is a group of automorphisms of $\mathcal{M}$. We consider the vector state $\omega(x) = (\Omega, x\Omega)$ $x \in \mathcal{M}$ with Ω a cyclic element of $\mathcal{H}$, such that $\Omega \in \mathcal{D}(H)$.

* Dedicated to Professor L. P. Bouckaert on the occasion of his seventieth birthday

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