Commun. math. Phys. 57, 161—164 (1977)

Communications in Mathematical Physics

Absence of Classical Lumps*

R. Weder**

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Abstract. Solutions of the equations of classical Yang-Mills theory in four dimensional Minkowski space are studied. It is proved (Theorem 1) that there is no finite energy (nonsingular) solution of the Yang-Mills equations having the property that there exists ε, R, $t_0 > 0$ such that

$$E_R(t) = \int_{|\bar{x}| \leqq R} \theta_{00}(t, \bar{x}) d^3\bar{x} \geqq \varepsilon \quad \text{for every} \quad t > t_0\,,$$

$\theta_{00}(\bar{x}, t)$ being the energy density. Previously known theorems on the absence of finite energy nonsingular solutions that radiate no energy out to spatial infinity are particular cases of Theorem 1. The result stated in Theorem 1 is not restricted to the Yang-Mills equations. In fact, it extends to a large class of relativistic equations (Theorem 2).

I. Introduction

In a very interesting paper [1] Coleman has proved that there are no (nontrivial) finite energy nonsingular solutions of classical Yang-Mills theory in four dimensional Minkowski space that do not radiate energy out to spatial infinity. More precisely he has proved:

Theorem *(Coleman). The only finite energy nonsingular solution of the Yang-Mills equations in four dimensional Minkowski space satisfying*

$$\lim_{|\bar{x}| \to \infty} |\bar{x}|^{3/2+\delta} F^a_{\mu\nu}(t, \bar{x}) = 0\,, \qquad \delta > 0 \tag{1}$$

uniformly in $|\bar{x}|$ *and* $t, t > 0$, *is the vacuum solution.* $F^a_{\mu\nu}(t, \bar{x})$ *is the field strength,* $\mu\nu = 0, 1, 2, 3$ *are space-time indexes and* a *is an internal index.* $\bar{x} \in \mathbb{R}^3$, $t \in \mathbb{R}$, *and the* $F^a_{\mu\nu}(t, \bar{x})$ *are real valued functions in* $\mathbb{R}^4$.

---

* Supported in part by the National Science Foundation under Grant PHY 75-21212

** On leave from the Universiteit Leuven, Belgium

Present address: Princeton University, Jadwin Hall, P.O. Box 708, Princeton, New Jersey 08540, USA