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Causality, Localizability, and Holomorphically Convex Hulls

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Abstract. A generalization of local commutativity to the fields with an exponential momentum-space growth $\sim e^{l\|p\|}$ is considered. To study the local properties of such fields we associate to each space-time region $\mathcal{O}$ a topology $\tau(\mathcal{O})$ on the test function space. It is shown that under any choice of the topology the fields of exponential growth are localizable only in space-time regions large in comparison with l. This happens because not any domain in the space of several complex variables is a domain of holomorphy. However, by specifying the topology through the use of holomorphically convex domains in $\mathbb{C}^4$, one can attach certain meaning to local commutativity for arbitrarily close spacelike separated regions of $\mathbb{R}^4$.

1. Introduction

A little more than 10 years ago Meyman [1] and Jaffe [2] showed that in the momentum representation the exponential growth of vacuum expectation values is physically marked, since beginning from this growth the fields change essentially their local properties. More precisely, there exist two natural boundaries. The first one characterizes the fields whose domain of definition contains test functions with compact support. This boundary was found by Jaffe with the aid of the theory of quasianalytic classes and looks like this

$$\int_1^\infty \frac{\ln g(t)}{t^2} dt < \infty . \tag{1}$$

Here g is a function characterizing the growth in p-representation. For example, the growth as $\exp\{\|p\|/(\ln\|p\|)^{1+\varepsilon}\}$ meets this requirement, whereas $\exp\{\|p\|/\ln\|p\|\}$ does not. Since in the test function space different operations are determined, the presence of only one function of compact support leads automatically to a rather large store of such functions. In particular, in the Jaffe spaces the following important property holds. For any pair of bounded space-time regions $\mathcal{O}_1$, $\mathcal{O}_2$ with disjoint closures one can find a test function which is equal to zero in $\mathcal{O}_1$ and to unity