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The Deterministic Version of the Glimm Scheme*

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Abstract. The Glimm scheme for solving hyperbolic conservation laws has a stochastic feature; it depends on a random sequence. The purpose of this paper is to show that the scheme converges for any equidistributed sequence. Thus the scheme becomes deterministic.

1. Introduction

We consider the initial value problem for the system of general conservation laws:

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=0, \quad t \geqq 0, \quad -\infty<x<\infty, \tag{1.1}$$

$$u(x,0)=u_0(x), \quad -\infty<x<\infty, \tag{1.2}$$

where u and $f(u)$ are n-vectors, and f is a smooth function of u. The system is assumed to be *strictly hyperbolic*, that is, the matrix $\partial f(u)/\partial u$ has real and distinct eigenvalues $\lambda_1(u)<\lambda_2(u)<\ldots<\lambda_n(u)$ with corresponding right eigenvectors $r_1(u)$, $r_2(u), \ldots, r_n(u)$. Since (1.1), (1.2) in general does not have smooth solution; we look for weak solution in the distributional sense. A bounded measurable function $u(x,t)$ is a weak solution if

$$\iint_{t \geqq 0}\left[u\frac{\partial \varphi}{\partial t}+f(u)\frac{\partial \varphi}{\partial x}\right]dx\,dt+\int_{t=0} u_0(x)\varphi(x,0)=0 \tag{1.3}$$

for any smooth function $\varphi(x,t)$ with compact support in $t \geqq 0$.

In [1], Glimm introduces a difference scheme for solving (1.1), (1.2). We now describe briefly the Glimm scheme. Choose any mesh lengths r, s, r/s bounded, which satisfy the Courant-Friedrich-Lewy condition:

$$\frac{r}{s} \geqq \max_{i=1,2,\ldots,n} |\lambda_i(u)|$$

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