Commun. math. Phys. 57, 111—116 (1977)

Communications in
Mathematical
Physics

Connection between the Spectrum Condition and the Lorentz Invariance of the Yukawa$_2$ Quantum Field Theory

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Abstract. We prove, under assumptions, the Lorentz invariance of some quantum field theories. In the separate paper we show that our assumptions are fulfilled in the (renormalized) Yukawa$_2$ quantum field theory with the periodic boundary conditions.

In the present paper we prove, under several assumptions, the Lorentz invariance of some quantum field theories. In a separate paper we show that our assumptions are fulfilled for the CPT invariant states in the (renormalized) Yukawa$_2$ quantum field theory with the periodic boundary conditions (see also [1]). Keeping this application in mind, we bound ourselves to the case of the Yukawa$_2$ quantum field theory.

For the sake of convenience, we introduce a space of four-component complex-valued test functions and we set for each four-component function

$$\Psi_0(h) := \sum_{\alpha=1}^{2} \int dx \psi_0^\alpha(x) h_\alpha(x) + \sum_{\alpha=3}^{4} \int dx \bar{\psi}_0^{\alpha-2}(x) h_\alpha(x)$$

where $\psi_0^\alpha(x)$ is the free fermion field in the one-dimensional space and $\overline{\psi_0}(x) = \psi_0(x)^* \gamma_0$. If $h(x)$ is a four-component function, we set

$$\mathrm{supp}\, h := \bigcup_\alpha \mathrm{supp}\, h_\alpha(x).$$

Let $\mathfrak{A}_f$ be the field algebra for the Y_2, as it is defined in [2], i.e., $\mathfrak{A}_f$ is the C^* algebra defined as the norm closure of $\bigcup_B \mathfrak{A}_f(B)$, where $\mathfrak{A}_f(B)$ is the von Neumann algebra generated by bounded functions of the time zero free scalar field $\varphi_0(f)$ and its time derivative $\pi_0(f)$, $f \in C_0^\infty(\mathbb{R})$, $\mathrm{supp} f \subset B$, and by the operators of the free fermion field $\Psi_0(h)$, $h \in C_0^\infty(\mathbb{R}) \otimes \mathbb{C}^4$, $\mathrm{supp}\, h \subset B$.

Let $\mathscr{P}_2$ be the Poincare group (= the inhomogeneous Lorentz group) in two space-time dimensions, i.e. the three-parameter group of transformations

$$\{a_0, a_1, \lambda\}(t, x) = (a_0 + \tfrac{1}{2}(\lambda^2 + \lambda^{-2})t + \tfrac{1}{2}(\lambda^2 - \lambda^{-2})x,$$
$$a_1 + \tfrac{1}{2}(\lambda^2 - \lambda^{-2})t + \tfrac{1}{2}(\lambda^2 + \lambda^{-2})x)\,.$$