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Quantum Detailed Balance and KMS Condition

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Abstract. A definition of detailed balance for quantum dynamical semigroups is given, and its close connection with the KMS condition is investigated.

1. Introduction

In recent works [1–3] various definitions of detailed balance for a quantum Markovian master equation have been proposed and discussed. In this paper, we give a definition of detailed balance for a quantum dynamical semigroup of a W^*-algebra, which extends the analogous notion proposed in [3] for quantum dynamical semigroups of matrix algebras (for a heuristic motivation, based on the analogy with the corresponding classical concept, see [3, 4]). We give the general form of the generator L of a dynamical semigroup of $\mathscr{B}(\mathscr{H})$ satisfying detailed balance and with a norm continuous dissipative part, thus extending the result of [3].

The physical meaning of this seemingly formal definition is investigated by showing that the property of detailed balance is characteristic of dynamical semigroups describing relaxation to thermal equilibrium, thus providing yet another characterization of KMS states.

2. Quantum Detailed Balance

Let $\mathscr{M}$ be a W^*-algebra. A *dynamical semigroup* of $\mathscr{M}$ [5–7] is a weakly *-continuous one-parameter semigroup $\{\Phi_t : t \geqq 0\}$ of completely positive identity preserving normal maps of $\mathscr{M}$ into itself, with Φ_0 the identity map.

Let ϱ be a faithful normal state on $\mathscr{M}$ which is stationary under $\{\Phi_t\}$, and denote by $(\mathscr{K}, \pi, \Omega)$ the GNS triple associated to ϱ. There exists [8, 9] a strongly continuous contraction semigroup $\{\hat{\Phi}_t\}$ on $\mathscr{K}$ such that

$$\hat{\Phi}_t \pi(A)\Omega = \pi(\Phi_t(A))\Omega \quad \text{for all} \quad A \in \mathscr{M}, \qquad t \geqq 0. \tag{2.1}$$

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