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Non-Equilibrium Dynamics of Two-dimensional Infinite Particle Systems with a Singular Interaction

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Abstract. The infinite system of Newton's equations of motion is considered for two-dimensional classical particles interacting by conservative two-body forces of finite range. Existence and uniqueness of solutions is proved for initial configurations with a logarithmic order of energy fluctuation at infinity. The semigroup of motion is also constructed and its continuity properties are discussed. The repulsive nature of interparticle forces is essentially exploited; the main condition on the interaction potential is that it is either positive or has a singularity at zero interparticle distance, which is as strong as that of an inverse fourth power.

1. Introduction

In this paper we extend some of our earlier results [3] on the existence of non-equilibrium dynamics of one-dimensional infinite particle systems to infinite systems of two-dimensional particles interacting by conservative repulsive forces of finite range. For a detailed motivation of this problem see [1–3], where further references are given on equilibrium dynamics as well.

Consider a finite or infinite system ω of two-dimensional particles. We assume that particles are numbered by a nonempty subset J of the set I of integers, the position and the velocity of the i-th particle, $i \in J$, will be denoted by x_i and v_i, respectively. Conservative two-body forces are given by the negative gradient $F = -\operatorname{grad} U$ of a symmetric real function $U = U(x)$ of two variables $(x^{(1)}, x^{(2)}) = x$, U is the interaction potential. For equal particles of unit mass indexed by $J \subset I$, Newton's equations of motion read formally as

$$\frac{dv_i}{dt} = -\sum_{j \in J_i} \operatorname{grad} U(x_i - x_j), \quad \frac{dx_i}{dt} = v_i \,; \quad i \in J \tag{NJ}$$

with initial conditions specifying the position and the velocity of each particle at time zero. The full system, when $J = I$, will be denoted as (NI),
$J_i = \{j\,; j \in J, j \neq i\}$ if $i \in J$.