

The Energy-Momentum Spectrum in the Yukawa₂ Quantum Field Theory

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Abstract. We prove that the Yukawa₂ quantum field theory with periodic boundary conditions satisfies the spectral condition, i.e., the joint spectrum of the energy operator H and the momentum operator P is contained in the forward cone. In addition, the φ -bound is obtained.

1. Introduction

In the present paper the Yukawa₂ ($= Y_2$) interaction in two-dimensional space time is considered. This theory provides an example of the theory which satisfies all the Haag-Kastler axioms and many of the Wightman and Osterwalder-Schrader axioms. In the present paper we prove the spectral condition for the Y_2 theory with the periodic boundary conditions. In addition, we obtain the uniform bounds on the boson field operators and on their derivatives with respect to coordinates.

The proof of the spectrum condition is divided into the following parts.

In Section 2 we prove the boundedness below and the $N_{\tau, \nu}$ estimate for the Lorentz rotated Hamiltonian in the Y_2 theory,

$$\beta_0 H_{V, \sigma} + \beta P_V + c_1(\beta, \tau) \geq c_2(\beta, \tau) N_{\tau, \nu}, \quad \tau < 1, \quad \beta_0^2 - \beta^2 = 1, \quad \beta_0 > 0$$

uniformly in the ultraviolet cut-off σ .

In Section 3 we prove the Osterwalder-Schrader positivity condition in the theory with $H_{0, \nu}(\beta) (= \beta_0 H_{0, \nu} + \beta P_V)$ as the Hamiltonian and P_V as the momentum operator.

In Section 4 we prove that the free vacuum $\Omega_{0, \nu}$ overlaps (see [1]) the vacuum for $H_V(\beta) = \beta_0 H_V + \beta P_V$.

In Section 5, using the Osterwalder-Schrader positivity in the spatial direction, we prove the main result, that the energy-momentum spectrum for the Y_2 interaction in a periodic box lies in the forward light cone. As a consequence, the uniform estimates on the derivatives of the boson field operators and the spectrum condition for the Y_2 interaction with the periodic boundary conditions follow (Section 6). In Section 6 the φ -bounds are also obtained.