

A Comment on the Symmetries of Kerr Black Holes

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Abstract. Conditions are given for the linear dependence of the two Killing vectors, found by Hughston and Sommers to exist in a class of Einstein-Maxwell fields of Petrov type *D*. The Killing tensors associated with these fields are shown to be contracted products of Killing-Yano tensors.

1. Introduction

In an elegant paper Hughston and Sommers [1] have shown how the symmetries of the Kerr black hole can be inferred by an argument centering around properties of the curvature tensor. They also show that an inference of this sort holds for the class of Petrov type *D* Einstein-Maxwell fields for which the electromagnetic field is non null with principal rays aligned with those of the Weyl tensor. If spinors o_A, i_A with $o_A i^A = 1$ are chosen so that the null vectors $o_A \bar{o}_{A'}$ and $i_A \bar{i}_{A'}$ are in the direction of these principal rays then the Maxwell spinor ϕ_{AB} can be written in terms of a single scalar field γ as

$$\phi_{AB} = \gamma^{2/3} o_{(A} i_{B)}. \tag{1.1}$$

Two further spinor fields X_{AB} and γ_{ABCD} are defined by

$$X_{AB} = \gamma^{-1/3} o_{(A} i_{B)} \tag{1.2}$$

and

$$\gamma_{ABCD} = \gamma o_{(A} o_B i_C i_{D)}. \tag{1.3}$$

These three fields satisfy the Maxwell equation, twistor equation and spin two rest mass field equation respectively, i.e.

$$\nabla^A_{A'} \phi_{AB} = 0 \tag{1.4}$$

$$\nabla^{A'}_{(A} X_{BC)} = 0 \tag{1.5}$$

and

$$\nabla^A_{A'} \gamma_{ABCD} = 0. \tag{1.6}$$