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Phase Space Cell Expansion and Borel Summability for the Euclidean φ_3^4 Theory

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Abstract. The stability of the free energy is proved for complex values of the coupling constant by the way of a convergent expansion. As a consequence, one obtains the Borel summability of the perturbation series.

I. Introduction

I.1.

Since the proof by Glimm and Jaffe [1] of the positivity of the φ^4 Hamiltonian in three dimensions, many other results have enlarged our knowledge on this model. Its present status is quite similar to that of $P(\varphi)_2$ theories ([2, 3]): the Wightman axioms with mass gap were proven by Feldman and Osterwalder [4] and Magnen and Sénéor [5] for the weak coupling region, the existence of phase transitions were shown by Fröhlich et al. [6], also Park has shown the convergence of the lattice approximation [7], and Burnap has investigated the particle structure [12]. To complete the analogy with two dimensional theories and in particular with the corresponding φ_2^4 theory, one would like to know the connection of the φ_3^4 theory with its perturbation expansion. From this arises the question of extending the proof of [1] to complex values of the coupling constant. On the other hand, a simplification of the proof given in [1] can allow a better understanding of the ideas introduced and their application to other types of problems. This article presents a contribution in these two directions. In particular, we solve the first problem and prove as a by-product the Borel summability of the theory and its uniqueness with respect to perturbation theory. It is left to the reader to judge if our modified proof of [1] is simple enough!

We want at this point to emphasize the fact that all the results quoted above were obtained using Euclidean space methods. The control of the φ_3^4 theory goes

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