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Phase Transitions for a Continuous System of Classical Particles in a Box*

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Abstract. A continuous classical system involving an infinite number of distinguishable particles is analyzed along the same lines as its quantum analogue, considered in [1]. A commutative C^*-algebra is set up on the phase space of the system, and a representation-dependent definition of equilibrium involving the static KMS condition is given. For a special class of interactions the set of equilibrium states is realized as a convex Borel set whose extremal states are characterized by solutions to a system of integral equations. By analyzing these integral equations, we prove the absence of phase transitions for high temperature and construct a phase transition for low temperature. The construction also provides an example of a translation-invariant state whose decomposition at infinity yields states that are not translation-invariant. Thus we have an example in the classical situation of continuous symmetry breaking.

Section 0. Introduction

We wish to study the equilibrium states of an infinite collection of distinguishable, classical-mechanical particles in a one-dimensional box, where the interaction is infinitely weak as in the quantum analogue considered in [1].

The first step is to single out a certain class of states p for which there exists a limiting derivation δ_p on the underlying algebra $\mathscr{A}$ of observables with range in $\pi_p(\mathscr{A})''$, where $(\mathscr{H}_p, \pi_p, \Phi_p)$ is the canonical cyclic representation of $\mathscr{A}$ with respect to p. The second step is to introduce a notion of equilibrium, and we do so by using an extension of the static KMS condition (which is studied in [2] for another classical situation) to the type of derivation considered here. We do not consider the dynamical KMS condition [3] because it is not clear whether the derivation δ_p generates a dynamics in the representation associated with p. The third step is to prove that the decomposition at infinity of an equilibrium state yields equilibrium states that are trivial at infinity, and this is done for a special class of interactions. The fourth step is to characterize the equilibrium states that are trivial at infinity as finite sets of functions satisfying the system (0.2) of equations, and, again, this is done for a special class of interactions. Using this explicit characterization we prove

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