Commun. math. Phys. 55, 293—298 (1977)

Communications in
Mathematical
Physics

An Inequality on S Wave Bound States, with Correct Coupling Constant Dependence

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Abstract. We prove that the number of S wave bound states in a spherically symmetric potential $gV(r)$ is less than

$$g^{1/2}\left[\int_0^\infty r^2 V^-(r)dr \int_0^\infty V^-(r)dr\right]^{1/4}$$

where V^- is the attractive part of the potential, in units where $\hbar^2/2M=1$.

I. Introduction

It is well known that in the limit of large coupling constants the number of S wave bound states in a potential $V(r)$ behaves asymptotically like [1], [2]

$$n(g) \simeq g^{1/2}\frac{1}{\pi}\int_0^\infty [V^-(r)]^{1/2}dr \tag{1}$$

where $V^-(r)$ is the attractive part of $V(r)$, in units such that $\hbar^2/2M=1$. This asymptotic theorem holds under various sufficient conditions. One of them is that $V(r)$ should be piecewise monotonous [1] with a finite number of monotony intervals. Another [2] is that $\int_0 [V^-(r)]dr$ converges and that V decreases fast enough at infinity. However, it is clearly impossible to turn the asymptotic equality (1) into a strict bound because bound states can easily be produced by delta function potentials; however the integral of the square root of a delta function is zero, crudely speaking. One way out is to require *monotony* of the potential, which excludes delta functions. Then one gets the Calogero bound [3]

$$n < g^{1/2}\frac{2}{\pi}\int_0^\infty [V^-(r)]^{1/2}dr \tag{2}$$

V^- monotonous decreasing.