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Non-Equilibrium Dynamics of One-dimensional Infinite Particle Systems with a Hard-Core Interaction

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Abstract. An infinite system of Newton's equation of motion is considered for one-dimensional particles interacting by a finite-range hard-core potential of singularity like an inverse power of distance between the hard cores. Existence of limiting solutions is proved for initial configurations of finite specific energy and the semigroup of motion is constructed if energy fluctuations near infinity increase only as a small power of distance from the origin. In this case uniqueness of solutions is also proved and the solution is a weakly continuous function of initial data. The allowed set of initial configurations carries a wide class of probability measures including Gibbsian fields with different potentials. In the absence of hard cores limiting solutions are constructed for initial configurations with a logarithmic order of energy and density fluctuations.

1. Introduction

The aim of this paper is to extend the results by Harris [1] and Lanford [2] on the existence of non-equilibrium dynamics of infinitely many particles on the line. The interaction is given by a translation invariant hard-core pair potential U of finite range, thus the interparticle force is just the negative derivative of U and the equations of motion are those of classical mechanics with this conservative force. As it is well known (see [1, 2, 5]) a solution to such an infinite system of differential equations exists only for a relatively small set of initial configurations, and the solution, if any, is not unique in the usual sense.

For the purposes of non-equilibrium statistical mechanics the semigroup of motion should be constructed in a set of configurations large enough to carry a class of Gibbsian fields. Let us remark that equilibrium dynamics that is a semigroup acting in the support of a Gibbsian field associated with the same potential as the dynamics itself has been constructed by several authors ([3–7]) while Lang [8] considers a first order system with additional white-noise terms. As regards non-equilibrium dynamics, in Harris' collision model (zero-range interaction) U is formally the δ-function, Lanford [2] treates the case of not necessarily symmetric,