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Irreversible Dynamics of Infinite Fermion Systems

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Abstract. We investigate the irreversible dynamics of infinite systems as specified by completely positive, strongly continuous, one-parameter semi-groups on a suitable C^*-algebra. Having shown how to construct such a semigroup from a fairly general evolution equation we determine when the semigroup is spatial with respect to a given representation of the algebra. A special class of exactly soluble evolution equations on the CAR algebra is studied in detail in order to test conjectured extensions of the theory.

§ 1. Introduction

If $\mathscr{A}$ is a C^*-algebra, which we shall always assume possesses an identity element 1, a dynamical semigroup on $\mathscr{A}$ is defined as a strongly continuous one-parameter semigroup of completely positive [12, p. 136; 47] identity-preserving maps T_t of $\mathscr{A}$ into itself. Such semi-groups arise in various contexts in non-equilibrium quantum statistical mechanics [12, 27], sometimes in the Heisenberg picture as above, and sometimes in the Schrodinger picture. They may be obtained in the weak or singular coupling limit when a system interacts with an infinite external reservoir [8, 13, 23, 35, 38]. In the converse direction given a dynamical semigroup one may frequently dilate it to a dynamical group, specified in some sense by a Hamiltonian, on a larger system [11, 20, 22, 33].

Two special types of dynamical semigroup are fairly well understood. In the first case [12], $\mathscr{A}$ is the algebra $\mathscr{L}(\mathscr{H})$ of all bounded operators on a separable Hilbert space $\mathscr{H}$ and T_t is obtained by duality from a one-parameter semigroup of the space of trace class operators on $\mathscr{H}$. In the second case [19, 30, 42], T_t is a one-parameter group of *-automorphisms of $\mathscr{A}$. Results known in one or other of these two special cases motivate much of the present work.

Associated with a dynamical semigroup T_t on $\mathscr{A}$ is its evolution equation

$$\frac{dX}{dt}=Z(X)$$