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Some Finite Dimensional Integrable Systems and Their Scattering Behavior*

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Abstract: We consider a class of Hamiltonian systems which posses integrals expressible in terms of the eigenvalues of some associate matrices. Moreover, these systems will be solved explicitly and their scattering behavior investigated using additional associated matrices whose eigenvalues change in time during a flow.

1. Introduction

We consider Hamiltonian systems of n particles on a line interacting with each other where the Hamiltonian is of the form:

$$H_\alpha(x, y) = \frac{1}{2}\sum_{i=1}^{n} y_i^2 + \sum_{1 \leqq i < j \leqq n} V(x_i - x_j) + \sum_{i=1}^{n} W(x_i). \tag{1.1}$$

The examples of such pairs of potentials $(V(x), W(x))$ to be considered are:

$$(x^{-2}, -\alpha^2 x^2/2), \tag{A}$$

$$\left(\left(\frac{1}{2}\coth x/2\right)^2, \alpha e^x\right). \tag{B}$$

Calogero and Marchioro [1] and Sutherland [2] have studied some of these potentials in the context of quantum mechanics, and their work suggested looking at the classical systems. For the case $\alpha = 0$, Moser [3] has shown that both of the above examples are integrable systems, i.e., possess n integrals whose associated Hamiltonian flows commute, and in addition the integrals are rational in (x_i, y_i), (e^{x_i}, y_i) respectively. The method he used was based on the isospectral technique of Lax [4], first applied by Flaschka [5] to the Toda lattice. This consists in the construction of a matrix function of (x, y) whose spectrum remains fixed in t if $x = x(t)$, $y = y(t)$ are solutions of the above Hamiltonian system. We then take the

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