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Correlation Inequalities and Equilibrium States

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Abstract. For an infinite dynamical system, idealized as a von Neumann algebra acted upon by a time translation implemented by a Hamiltonian H, we characterize equilibrium states (KMS) by stationarity, a Bogoliubov-type inequality and continuous spectrum of H, except at zero.

§ 1. Introduction

The equilibrium states of a finite volume system in statistical mechanics is usually given by the Gibbs-ensembles.

To describe bona fide physical phenomena it is well known that one has to take the so-called thermodynamic limit i.e. the volume tending to infinity, of any of the Gibbs ensembles. These "limit Gibbs' states" have an interesting property, they satisfy the so-called KMS-condition [1, 2].

In [3] Roepstorff derived a stronger version of the Bogoliubov inequality [4] for Gibbs states (for KMS-states see [5]).

Let $\langle . \rangle_{\beta H}$ denote the thermal average with respect to the Hamiltonian H and the inverse temperature $\beta = 1/kT$. For any pair of observables x, y the scalar product $(.,.)_\sim$ is defined by:

$$(x, y)_\sim = \frac{1}{\beta} \int_0^\beta d\lambda \langle \exp(\lambda H) x^* \exp(-\lambda H) y \rangle_{\beta H}$$

(see also [6]). In [3] the following inequality is derived

$$(x, x)_\sim \leqq [\langle x x^* \rangle_{\beta H} - \langle x^* x \rangle_{\beta H}] / \ln \langle x x^* \rangle_{\beta H} / \langle x^* x \rangle_{\beta H} . \tag{1}$$

Of course we have not to insist on the importance of the Bogoliubov inequality and its stronger version in statistical mechanics (see e.g. [7]).

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