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There Are No Classical Glueballs*

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Abstract. I show that there are no finite-energy non-singular solutions of classical Yang-Mills theory in four-dimensional Minkowski space that do not radiate energy out to spatial infinity. Finite-energy non-singular solutions that are either static or periodic in time, are a fortiori non-radiant; thus this generalizes earlier theorems that state that there are no such solutions.

1. Introduction

In a linear classical field theory, like free electrodynamics, any non-singular initial configuration of fields of finite energy will eventually spread out over all space; whatever the initial configuration, the final state is simply outgoing radiation. In contrast, certain non-linear field theories are known to have finite-energy non-singular solutions that can be described as lumps of energy held together by their own self-interaction; no energy is radiated to spatial infinity [1]. For brevity, I will call such solutions "lumps". I emphasize that lumps, as I define them, may have arbitrary time-dependence, as long as they do not radiate away any of their energy.

In this note I show that classical Yang-Mills theory in four-dimensional Minkowski space has no lumps. This extends the known theorems that state that this theory possesses no finite-energy non-singular solutions that are either time-independent [1, 2] or periodic in time [3].

If Yang-Mills lumps had existed, they would have been classical analogues of the "glueballs" of quantum chromodynamics, colorless bound states composed exclusively of gauge-field quanta ("gluons"). Of course, the nonexistence of classical glueballs says nothing against the existence of quantum glueballs, anymore than the instability of the classical Hydrogen atom says anything against the existence of the quantum Hydrogen atom. However, it does indicate that it is unlikely that insight into the structure of quantum glueballs will be obtained by studying the classical limit.

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