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In One and Two Dimensions, Every Stationary Measure for a Stochastic Ising Model is a Gibbs State

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Abstract. It is shown that one and two dimensional (generalized) stochastic Ising models with finite range potentials have only Gibbs states as their stationary measures. This is true even if the stationary measure or the potential is not translation invariant. This extends previously known results which are restricted to translation invariant stationary measures and potentials. In particular if the potential has only one Gibbs state the stochastic Ising Model must be ergodic.

0. Introduction

One of the unpleasant aspects of studying Gibbs states with stochastic Ising models is that one does not know, in general, if uniqueness of phase implies that any associated stochastic Ising model is ergodic. The converse statement, namely that ergodicity of some associated stochastic Ising model implies uniqueness of phase, is well-known and is one of the major reasons for the interest in stochastic Ising models (cf. [5] for example). The purpose of the present paper is to show that in one and two dimensions every stationary measure for any reasonable stochastic Ising model associated with a given potential energy function is a Gibbs state for that potential. In particular, this result resolves the question mentioned above, at least in one and two dimensions.

The technique on which our proof turns is that of considering how the free energy of a state evolves under the action of the stochastic Ising model. This idea has been used before in the study of similar questions (cf. [1], [2], and [7]). In these earlier applications, the free energy functional was used as a kind of Liapounov function. Unfortunately, as a Liapounov function the free energy functional isn't completely satisfactory unless everything, including the initial state, is assumed to be translation invariant. Our application of the free energy functional is quite different. In particular, we will be concerned with the free energy of a state in a finite

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