Commun. math. Phys. 54, 293—297 (1977)

Communications in
Mathematical
Physics

Irreducible Quantum Dynamical Semigroups

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Abstract. We study some irreducible and ergodic properties of quantum dynamical semigroups, and apply our methods to semigroups of Lindblad type.

§ 1. Introduction

There has been great interest recently in the study of dynamical semigroups in quantum systems as semigroups on operator algebras with certain positivity properties such as complete positivity, see for example [3, 4, 6–8, 11, 12, 14]. Here we study some irreducible and ergodic properties of such semigroups, with particular reference to locally completely positive maps.

In § 2 we prove an unbounded version of Lindblad's result relating dissipations and semigroups of locally completely positive maps on C^*-algebras. In the third section we introduce a concept of an irreducible process, which is weaker than that considered by Davies [3] in the Schrödinger picture. We show that a dynamical semigroup of locally completely positive maps on a W^*-algebra is irreducible if and only if the largest W^*-algebra in the fixed point set is trivial. In § 4 we apply our results to semigroups of completely positive maps of Lindblad type [3, 4, 11, 12], and discuss the relationship of our work with that of [3]. In particular we gain more insight into Davies' result on "doubly stochastic" quantum processes [3, Theorem 19]. Some weaker results in these directions have also been obtained by Spohn for completely positive dynamical semigroups of N-level systems by entirely different methods [14].

It is a pleasure to thank E. Størmer for his kind hospitality at the University of Oslo, and for helpful comments with the presentation of the manuscript. The author is also grateful to E. B. Davies for arousing his interest in the problem by introducing him to the work of H. Spohn.

§ 2. Dissipations and Locally Completely Positive Semigroups

Definition 2.1. A linear map Φ between C^*-algebras $\mathscr{A}$ and $\mathscr{B}$ is said to be locally completely positive if it satisfies the Kadison-Schwarz inequality:

$$\|\Phi\| \Phi(a^* a) \geqq \Phi(a)^* \Phi(a) \quad \text{for all} \quad a \text{ in } \mathscr{A}.$$