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Holonomic Character and Local Monodromy Structure of Feynman Integrals

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Abstract. We prove that the micro-local holonomic structure controls the local monodromy structure of functions involved. This result plays an essential role in investigating "hierarchical principle" in perturbation theory.

§ 1. Introduction

At the occasion of Kyoto symposium ($M \cap \Phi$) in 1975, Sato [15] emphasized the importance of holonomic systems of micro-differential equations[1] in investigating the S-matrix and related functions. At least in the case of Feynman integrals this point of view (i.e. the use of over-determined system of linear differential equations) was also emphasized by Regge [12] as early as 1967.

In this approach, the first thing to do is to establish the fact that the S-matrix and/or Feynman integrals satisfy some holonomic systems of (micro-)differential equations. Partial results were given for Feynman integrals by Barucchi and Ponzaro [1] and Sato [15] and for the S-matrix by Kawai and Stapp [5, 6]. In this direction a decisive result has recently been given for arbitrary Feynman integrals by Kashiwara and Kawai [3, 4].

Having this situation in mind, we show in this article how the holonomic structure controls the local sheet structure of Feynman integrals. More precisely, we show in Theorem 2 that our main result (Theorem 1) applied to Feynman integrals entails that the local monodromy structure of the Feynman integral associated with a Feynman diagram D controls that of the Feynman integral associated with the "daughter" diagram D' of D under moderate conditions. Thus our results find an intimate connection with the celebrated "hierarchical principle" proposed by

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1 In Sato et al. [16], a holonomic system is called a maximally overdetermined system and a micro-differential equation (operator, resp.) is called a pseudo-differential equation (operator, resp.). Here we change our terminology according to the suggestion of Prof. Sato. We also use the terminology "holonomicity of a function" to indicate the holonomic character of the function, i.e., the fact that the function satisfies a holonomic system of (micro-)differential equations.