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Stationary Non-Equilibrium States of Infinite Harmonic Systems

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Abstract. We investigate the existence, properties and approach to stationary non-equilibrium states of infinite harmonic crystals. For classical systems these stationary states are, like the Gibbs states, Gaussian measures on the phase space of the infinite system (analogues results are true for quantum systems). Their ergodic properties are the same as those of the equilibrium states: e.g. for ordered periodic crystals they are Bernoulli. Unlike the equilibrium states however they are not "stable" towards perturbations in the potential.

We are particularly concerned here with states in which there is a non-vanishing steady heat flux passing through "every point" of the infinite system. Such "superheat-conducting" states are of course only possible in systems in which Fourier's law does not hold: the perfect harmonic crystal being an example of such a system. For a one dimensional system, we find such states (explicitely) as limits, when $t \to \infty$, of time evolved initial states μ_i in which the "left" and "right" parts of the infinite crystal are in "equilibrium" at different temperatures, $\beta_L^{-L} \neq \beta_R^{-1}$, and the "middle" part is in an arbitrary state. We also investigate the limit of these stationary ($t \to \infty$) states as the coupling strength λ between the "system" and the "reservoirs" goes to zero. In this limit we obtain a product state, where the reservoirs are in equilibrium at temperatures β_L^{-1} and β_R^{-1} and the system is in the unique stationary state of the reduced dynamics in the weak coupling limit.

1. Introduction

Our theoretical understanding of the properties of large, macroscopic size, objects is based to a great extent on the study of idealized model systems. Such models are particularly useful when it is possible to identify explicitely some observed behavior characteristic of macroscopic systems with properties of the models which appear,

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