Commun. math. Phys. 54, 81—96 (1977)

Communications in
Mathematical
Physics

Stationary Solutions of the Bogoliubov Hierarchy Equations in Classical Statistical Mechanics. 2

B. M. Gurevich

Laboratory of Statistical Methods, Moscow State University, Moscow, USSR

Iu. M. Suhov*

Centre de Physique Théorique, CNRS, Marseille,
and UER Marseille-Luminy, Université d'Aix-Marseille II, F-Marseille, France

Abstract. In the preceding paper under the same title we have formulated a theorem which describes the set of states (i.e., probability measures on phase space of an infinite system of particles in R^ν) corresponding to stationary solutions of the BBGKY hierarchy. We have proved the following statement: if G is a Gibbs measure (Gibbs random point field) corresponding to a stationary solution of the BBGKY hierarchy, then its generating function satisfies a differential equation which is "conjugated" to the BBGKY hierarchy. The present paper deals with the investigation of the "conjugated" equation for the generating function in particular cases.

0. Preliminaries

This paper is the second part of a work of the authors published under the same title. The first part of the work is [1]. All references to the paper [1] are marked by the index I: Theorem 1, I, condition (G_1, I), formula (2.2, I), etc. In paper [1] we have formulated the main theorem which describes all stationary solutions of the Bogoliubov hierarchy equations (B.h.e.) in a class of probability measures on phase space. The proof of this theorem is naturally divided into two parts. In [1] we have proved a statement (Theorem 1, I) which is, in a sense, the first part of the main theorem. Namely, we have showed that under some restrictions [see conditions (I_1, I)–(I_4, I) and (G_1, I)–(G_6, I)], the generating function of a Gibbs random field corresponding to a stationary solution of the B.h.e. satisfies a differential equation[1] [see (2.8, I)] which may be considered as a conjugate equation to the B.h.e. The present paper deals with the proof of the second part of the main theorem.

The second part of the main theorem is formulated in [1] as an assertion (Theorem 2, I) according to which any function satisfying the Equation (2.8, I) [and conditions (G_1, I)–(G_6, I)] has the form (2.7, I), i.e. generates an equilibrium state (in

* Permanent address: Institute of Problems of Information Transmission, USSR Academy of Sciences, Moscow, USSR

[1] Or, if one prefers, a system of differential equations