Commun. math. Phys. 54, 69—79 (1977)

Communications in Mathematical Physics

The Dynamical Instability of Nonrelativistic Many-Body Systems*

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Abstract. From computations in an exactly solvable many-body dynamical model we argue that, quite generally, a nonrelativistic quantum mechanics of infinitely many interacting particles must admit states without a global time evolution; equivalently, that the (quasi-local) observables of any such theory are not preserved in time by the Heisenberg dynamics. Our analysis is based on a dynamical instability common to interacting finite-particle systems.

1. Introduction

The dynamics of many-body systems is known to be less singular in (nonrelativistic) quantum mechanics than in (nonrelativistic) classical mechanics. For example it remains a famous unsolved problem in classical mechanics [1] whether or not there is a global time evolution for each initial state in the N-body coulomb problem ($N \geqq 4$ point particles in three dimensions with attractive coulomb interaction) excluding the set, of measure zero [2], of those initial states which lead to collisions. Yet such difficulties are smoothed out in quantum mechanics by the wavefunction formalism. For any interaction given by a Kato potential, a class that includes the coulomb interaction [3], it has been proven [4] that *every* physical initial state has a global time evolution which remains physical in time. (By a "physical" state we understood one in which the usual particle observables, such as position, momentum and energy, are well defined.) Thus there can be no "catastrophe", or breakdown in the time evolution, of *any* initial state in a quantum mechanics of the most important dynamical models.

However the above argument does not preclude the phenomenon of "dynamical instability" for N-body quantum models, that is, a *potential* breakdown in the dynamics which gets more severe as larger N are considered, which is manifested by an *actual* breakdown in the dynamics of the associated infinite-particle models (and is thus qualitatively as significant as a breakdown for large enough N).

* Work supported in part by the National Science Foundation

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