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On the Decay of Correlations in $SO(n)$-symmetric Ferromagnets*

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Abstract. We prove that for low temperatures T the spin-spin correlation function of the two-dimensional classical $SO(n)$-symmetric Ising ferromagnet decays faster than $|x|^{-\mathrm{const}\,T}$ provided $n \geqq 2$. We also discuss a nearest neighbor continuous spin model, with spins restricted to a finite interval, where we show that the spin-spin correlation function decays exponentially in any number of dimensions.

I. Introduction and Results

The Mermin-Wagner theorem [1] states that at non-zero temperatures the two dimensional Heisenberg model has no spontaneous magnetization. Consequently the spin-spin correlation function decays to zero at large distances, although the Mermin-Wagner theorem gives no indication of the rate of decay. Similar results apply for the classical $SO(n)$-symmetric ($n \geqq 2$) nearest neighbor Ising ferromagnets which we study here, see for example the paper of Mermin [2]. We establish a polynomial upper bound for the decay rate of the spin-spin correlation function for these models at very low temperatures. Fisher and Jasnow [3] have previously obtained a $\log^{-1}|x|$ decay.

To describe the $SO(n)$-symmetric ferromagnet, we consider the infinite lattice of unit spacing with sites labelled by indices $i \in \mathbb{Z}^2$. To each site i we associate an n-component classical spin s_i of unit length, $\|s_i\| = 1$. The spin-spin correlation function at inverse temperature $\beta = T^{-1}$ is

$$\begin{aligned} \langle s_0 \cdot s_x \rangle(\beta) &= Z^{-1} \prod_i \int d\Omega_i^{(n)} e^{\beta \sum_{\langle i,j \rangle} s_i \cdot s_j} s_0 \cdot s_x\,, \\ Z &= \prod_i \int d\Omega_i^{(n)} e^{\beta \sum_{\langle i,j \rangle} s_i \cdot s_j}\,, \end{aligned} \tag{1}$$

where $\sum_{\langle i,j \rangle}$ denotes a sum over nearest neighbor pairs, $\Omega_i^{(n)}$ is the invariant measure

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