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On the Type of Local Algebras in Quantum Field Theory

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Abstract. We give a simple sufficient condition for a von Neumann algebra to be Type III and apply it to some classes of algebras in QFT. For dilatation invariant local systems in particular we find that all sufficiently regular local algebras are Type III.

I. Introduction

An old problem of algebraic quantum field theory is the question of how far the axioms of Haag Araki determine the algebraic structure of local algebras. It is known by now that local algebras are of infinite type [1, 2], that certain types of nonlocal ones are Type III [3] resp. Type III_1 factors [4], and that in theories which are invariant with respect to dilatations of space time local algebras cannot be Type I in general [5].

The aim of this paper is to prove the Type III property for larger classes of algebras. In Section 2 we give a simple sufficient condition for a von Neumann algebra to be Type III which in Section 3 will be applied to various situations in QFT. For dilatation invariant theories in particular we shall get Type III for all local algebras belonging to not too irregularly shaped regions. However, since our argument involves short and long distance behaviour we cannot conclude that theories with well defined Gell Mann Low limit will have the same property without further assumptions as was argued in [5] for the case involving only the short distance behaviour.

Our arguments, however, lend support to the ancient conjecture that Type III should be a general feature of local algebras in QFT, a conjecture which up to now was apparently only based on the explicit computation of the type for the free field algebras [6].

II. The Type Theorem

Theorem. *Let $\mathscr{H}$ be a separable Hilbert space and $\mathscr{M}$ a von Neumann algebra in $\mathscr{L}(\mathscr{H})$ with a separating vector Θ; let $\mathscr{M}_1 \subset \mathscr{M}$ be a subalgebra of infinite type and*

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