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Schrödinger Operators with L^p_{loc}-Potentials

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Abstract. We discuss the question of when the closure of the Schrödinger operator, $-\Delta+V$, acting in $L^p(R^l, d^l x)$, generates a strongly continuous contraction semigroup. We prove a series of theorems proving the stability for $-\Delta: L^p \to L^p$ of the property of having a m-accretive closure under perturbations by functions in $L^q_{\text{loc}}(1<p\leqq q)$. The connection with form sums and the Trotter product formula are considered. These results generalize earlier results of Kato, Kalf-Walter, Semenov and Beliy-Semenov in that we allow more general local singularities, including arbitrary singularities at one point, and arbitrary growth at infinity. We exploit bilinear form methods, Kato's inequality and certain properties of infinitesimal generators of contractions.

1. Introduction and Results

Kato [1] showed that the L^2-operator sum, $-\Delta+V$, is essentially self-adjoint on $C_0^\infty(R^l)$ if $0\leqq V\in L^q_{\text{loc}}(R^l, d^l x)$, $q=2$. In particular, the Trotter product formula holds in this case. However, if $q<2$, it can happen that $\mathscr{D}(-\Delta)\cap\mathscr{D}(V)=\{0\}$, so that the operator sum $-\Delta+V$ is not densely defined. Nevertheless, in Semenov [3] and Beliy-Semenov [7, 12], an operator H is constructed so that the Trotter product formula

$$e^{-tH}=s\text{-}\lim(e^{-tH_0/n}e^{-tV/n})^n$$

holds so long as $0\leqq V\in L^q(R^l, d^l x)$ $q\geqq 1$ or $0\leqq V\in L^q_{\text{loc}}(R^l, d^l x)$, $q\geqq 1$. (Here and below the symbol s-lim stands for an L^2 strong limit.) H was constructed as a form sum, and, in the second case, Kato's inequality was essentially employed. In addition we developed a criterion for a sum to have an m-accretive closure.

We recall that an operator A is called m accretive if and only if $-A$ generates a contraction semigroup e^{-tA}. We call D an m-accretive core for A if and only if the closure of $A\restriction D$ is m-accretive. If more than one Banach space is possible, e.g. $D=C_0^\infty(R^l)$ we will sometimes modify the phase m-accretive with a Banach space, e.g. L^p-m-accretive.