

The Critical Behavior of ϕ_1^4

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Abstract. The eigenvalues, eigenfunctions, and Schwinger functions of the ordinary differential operator

$$H(\lambda, m) = \frac{1}{2}\{p^2 + \lambda q^4 + (m^2 - \lambda m^{-1})q^2\}$$

are studied as $\lambda \rightarrow \infty$. It is shown that the scaling limit of the Schwinger functions equals the scaling limit of a one dimensional Ising model. Critical exponents of $H(\lambda, m)$ are shown to equal critical exponents of the Ising model, while critical exponents of the renormalized theory are shown to agree with those of a harmonic oscillator.

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1. Introduction

The purpose of this paper is to explain the behavior of the eigenvalues, eigenfunctions, and Schwinger (or correlation) functions of the ordinary differential operator

$$\frac{1}{2}\{p^2 + \lambda q^4 + (m^2 - \lambda m^{-1})q^2\} \tag{1.1}$$