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Infinitesimal Discontinuities in Initially Stressed Relativistic Elastic Solids

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Abstract. In this paper we examine the influence of a general initial state of stress upon the propagation of infinitesimal (or weak) discontinuities in nonlinear relativistic elasticity. This influence, which materializes in alterations in the wave speeds, the general nonseparability in longitudinal and transverse waves, and the growth of the amplitude of infinitesimal discontinuities so as to form shock fronts, is first studied on the basis of a model of relativistic elasticity called hypoelasticity of zeroth order. The analytical treatment, however, is manageable only for the case of principal wave fronts for which the spatial direction of propagation coincides with a principal direction of the initial state of stress and, consequently, the wave fronts separate into longitudinal and transverse ones. Such notions as those of apparent elasticity moduli appear naturally in the analysis. Then a model of thermodynamical relativistic elasticity, referred to as neo-Hookean elasticity, is shown to be representable, insofar as wave-front propagation is concerned, by a special model of hypoelasticity of first order. The qualitative results obtained before concerning the influence of initial stresses are shown to apply equally to this description.

1. Introduction

The interest of relativistic elasticity for the study of the deformation of massive stellar objects on the one hand [1—4], and for a coherent approach to the vibrations of elastic detectors of gravitational waves on the other hand [5, 6], has been emphasized in recent years. The recent history of relativistic elasticity offers two avenues of development, one initiated by Synge [7], and the other that makes use of thermodynamical arguments and is illustrated, for instance, by the already quoted papers. In formulating his pioneering theory of relativistic elasticity, Synge sought to avoid the definition of an initially stress-free state which, because of the ever operating gravitational field, cannot exist. However, recent developments in relativistic continuum mechanics have placed in evidence some shortcomings of his formulation (no relation to thermodynamics, noninvariance of the constitutive equations with respect to the observers, i.e., "nonobjectivity"). Furthermore, it