Commun. math. Phys. 53, 221—231 (1977)

Communications in Mathematical Physics


# Inequalities on the Number of Bound States in Oscillating Potentials

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**Abstract.** For short-range oscillating potentials $V(r)$, such that $W(r) = -\int_r^\infty V(r')dr'$ possesses some regularity properties we establish inequalities on the number of bound states. In particular we show that by replacing $V(r)$ by $-4(W(r))^2$ in the classical inequalities we get bounds for this new class of potentials. Optimal bounds are also obtained. The behaviour for large coupling constants is studied.

## 1. Introduction and Outline

The purpose of this paper is to obtain upper bounds for the number of bound states of a class of spherically symmetric potentials introduced recently by Baeteman and one of us (K.C.) [1]. These potentials, although very singular and oscillating near the origin, are perfectly regular from the point of view of quantum scattering theory. In fact, it was even shown that the usual description of scattering via the Jost function applies to this class as well, without any modification, and therefore that this class generalizes the notion of regular potential. A further generalization has been made to non-spherical potentials by Combescure and Ginibre [2].

In this paper, we are concerned with the spherically symmetric case, and assume always that the potential has a short range, i.e., it decreases faster than $r^{-2}$ at large distances. More precisely, we assume that $V(r)$ is real, locally integrable away from the origin, and that

$$\int_a^\infty r|V(r)|dr < \infty\,, \qquad \forall a > 0\,. \tag{1}$$

Traditionally, regular spherically symmetric potentials were those for which $rV$ is absolutely integrable at the origin, i.e.,

$$\int_0^\infty r|V(r)|dr < \infty \tag{2}$$

[a possible, slightly different version is $\lim_{r\to 0} r^2|V(r)| = 0$].