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On the Equivalence between KMS-States and Equilibrium States for Classical Systems

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Abstract. It is shown that for any KMS-state of a classical system of non-coincident particles, the distribution functions are absolutely continuous with respect to Lebesgue measure; the equivalence between KMS states and Canonical Gibbs States is then established.

1. Introduction

In classical statistical mechanics macroscopic systems are described by "states" defined as probability measures on the phase space of the system. For infinite systems (the precise mathematical analogs of macroscopic systems) these measures are specified in terms of "local" distributions, i.e. n-particle distributions on bounded regions $\Lambda \subset \mathbb{R}^\nu, \mu_\Lambda^{(n)}$ [1]. Moreover, systems at equilibrium are specified by states satisfying certain conditions (e.g. D.L.R. equations $\equiv$ Gibbs states, K.-S.-equations, limit of finite volume grand-canonical states, variational principles). These conditions have been proved to be equivalent in many different cases [2–5]; however, in these proofs, it has always been assumed that the $\mu_\Lambda^{(n)}$ are absolutely continuous with respect to Lebesgue measure. This assumption gives thus a special status to the Lebesgue measure which has not been derived from physical principles.

On the other hand, it has been suggested that equilibrium states could also be defined as states satisfying the KMS-*condition;* in fact it is well known that for infinite quantum systems, Gibbs states are characterized by this KMS condition [6]; for infinite classical systems, Gallavotti and Verboven gave in a recent work [7] some sufficient conditions for the equivalence of the KMS-conditions and the Kirkwood-Salzburg equations; among those fairly strong conditions were in particular the conditions of absolute continuity of the $\mu_\Lambda^{(n)}$ with respect to Lebesgue measure, low density, exponential clustering, smoothness properties, ...

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