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Scattering Theory and Quadratic Forms: On a Theorem of Schechter*

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Abstract. We show how to extend Cook's method to a class of pairs whose difference is only a quadratic form.

This note is connected with the existence question for generalized wave operators

$$\Omega^{\pm}(A,B) = \underset{t\to\mp\infty}{s\text{-lim}}\, e^{itA}e^{-itB}P_{ac}(B),$$

where A and B are self-adjoint operators on a Hilbert space, $\mathscr{H}$, and $P_{ac}(B)$ is the projection onto $\mathscr{H}_{ac}(B)$, the set of vectorswhose spectral measure relative to B is absolutely continuous with respect to Lebesgue measure. Nearly twenty years ago, Cook [2] proved a result whose abstract form says

Theorem 1. *Let A, B be self-adjoint operators and suppose that there is a subset $\mathscr{D}$ of $D(B)$ dense in $\mathscr{H}_{ac}(B)$ so that for any $\varphi\in\mathscr{D}$, there is a T_0 with $e^{itB}\varphi\in D(A)$ for $|t|>T_0$ and*

$$\int_{|t|\geqq T_0} \|(B-A)e^{itB}\|\,dt<\infty\,. \tag{1}$$

Then $\Omega^{\pm}(A,B)$ exist.

Cook's theorem is widely applicible (see e.g. [6]) and it has the advantage of having an extremely simple proof. As regards existence alone, its main disadvantage is that it does not accomodate operators defined as sum of forms, e.g. $B=-\Delta$, $A=-\Delta+V$. If $(1+|x|)^{+1+\varepsilon}V\in L^{3/2}(R^3)+L^{\infty}(R^3)$, one can define A as a form sum and expects $\Omega^{\pm}(A,B)$ to exist. Indeed, this has been proven [5], but only by developing the rather elaborate machinery of smoothness [4] and weighted L^2 estimates [1, 3] *and the proof doesn't work in the multiparticle case.* For many years, there have been no improvement in results based on an estimate like (1), but Schechter [7] has recently proven:

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