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A Stronger Version of Bogoliubov's Inequality and the Heisenberg Model

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Abstract. On the basis of general inequalities in quantum statistical mechanics we derive a rigorous upper bound for the magnetization in the ferromagnetic quantum Heisenberg model with arbitrary spin and dimension $n \geqq 3$.

1. Introduction

Recently, Dyson et al. [1] proved the existence of a phase transition at non-zero temperature for the Heisenberg model with nearest neighbor coupling. The proof essentially relies on some new inequalities involving two-point functions. Some of these inequalities are quite general and, therefore, apply to any quantum system in thermal equilibrium. Others rest on the specific structure of the model (spin system, simple cubic lattice, nearest neighbor coupling etc.) and have limited applicability.

Our concern here will be with general estimates of the type used in [1–4]. One of these estimates will turn out to be an improved version of Bogoliubov's inequality [5] which proved to be a powerful tool in many cases. Recall, for instance, that Mermin and Wagner [6] used it to rule out a spontaneous ferromagnetic ordering for the Heisenberg model in one or two dimensions. We shall rederive this result using the stronger inequality and, applying the same argument to three and more dimensions, we shall obtain an upper bound for the reduced magnetization m:

$$0 \leqq m \leqq M < 1 .$$

In the zero external field limit and for a spin 1/2 lattice the bound $M(\beta)$ is implicitly given by the equation

$$1 - M = 2M(2\pi)^{-n} \int d^n p (e^{\beta E_p / M} - 1)^{-1} , \tag{1}$$

where β is the inverse temperature and E_p stands for the energy of a spin wave with momentum p. The integration is carried over the first Brillouin zone. By inspection, the result compares with that of the magnon approximation [7, 8]:

$$1 - m = 2(2\pi)^{-n} \int d^n p (e^{\beta E_p} - 1)^{-1} .$$