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The Cluster Expansion for Potentials with Exponential Fall-off*

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Abstract. Continuing the work of a previous paper, the Glimm-Jaffe-Spencer cluster expansion from constructive quantum field theory is adapted to treat quantum statistical mechanical systems of particles interacting by potentials that fall off exponentially at large distance. The Hamiltonian H_0+V need be stable in the extended sense that $H_0+4V+BN\geqq 0$ for some B. In this situation, with a mild technical condition on the potentials, the cluster expansion converges and the infinite volume limit of the correlation functions exists, at low enough density. These infinite volume correlation functions cluster exponentially. A natural system included in the present treatment is that of matter with the r^{-1} potential replaced by $e^{-\alpha r}/r$. The Hamiltonian is stable, but the system would collapse in the absence of the exclusion principle—the potential is unstable. Therefore this system cannot be handled by the classic work of Ginibre, which requires stable potentials.

1. Introduction

In a previous paper, [1], we adapted the Glimm-Jaffe-Spencer cluster expansion [8] to treat quantum statistical mechanical systems with finite range potentials. We now extend this program to include potentials that fall off exponentially. Under very general conditions we will obtain the infinite volume limit of correlation functions (in the Euclidean region) and their exponential clustering, at low density. We will later remark on some extensions of the present work to even more general potentials.

Matter (positive charged particles and negative charged identical fermions interacting with a r^{-1} potential) with the r^{-1} modified to $e^{-\alpha r}/r$, one of our matter-like systems, has been our main motivational example. For this system the Hamiltonian is stable; proofs of stability for the matter system [4, 5, 10] may be modified to show this. But the potential is not stable, [11] and in fact the system

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