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Asymptotic Behavior of Solutions to Certain Nonlinear Schrödinger-Hartree Equations*

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Abstract. The asymptotic behavior of solutions to the Cauchy problem for the equation

$$i\psi_t = \tfrac{1}{2}\Delta\psi - v(\psi)\psi\,, \quad v = r^{-1} * |\psi|^2\,,$$

and for systems of similar form, is studied. It is shown that the norms

$$\|\psi(t)\|^2_{L_2(|x| \leqq R)} + \|\nabla\psi(t)\|^2_{L_2(|x| \leqq R)}$$

are integrable in time for any fixed $R > 0$, from which it follows that

$$\lim_{t \to \infty} \|\psi(t)\|_{L_2(|x| \leqq R)} = 0\,.$$

Nevertheless, it is established that an L_2-scattering theory is impossible.

Introduction

We consider classical solutions to the Cauchy problem for the equations

$$\begin{aligned} &i\psi_t = \tfrac{1}{2}\Delta\psi - v(\psi)\psi \qquad (x \in \mathbb{R}^3,\ t > 0) \\ &v(\psi) = r^{-1} * |\psi|^2 = \int_{\mathbb{R}^3} |x-y|^{-1} |\psi(y,t)|^2 dy \qquad (r = |x|) \end{aligned} \tag{1}$$

and

$$i\partial\psi_j/\partial t = \tfrac{1}{2}\Delta\psi_j - \sum_{k=1}^{N} (\psi_j v_k - \psi_k v_{jk}) \qquad (j = 1, 2, \ldots, N) \tag{2}$$

where

$$v_{jk} = r^{-1} * \psi_j \overline{\psi_k}\,, \qquad v_k = v_{kk} = r^{-1} * |\psi_k|^2\,.$$

Equations (1), (2) are Coulomb-free versions of the time-dependent Hartree and Hartree-Fock equations. In [2] we have treated the existence question for

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