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Duality for Dual Covariance Algebras

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Abstract. One way of generalizing the definition of an action of the dual group of a locally compact abelian group on a von Neumann algebra to non-abelian groups is to consider $\mathscr{L}(G)$-comodules, where $\mathscr{L}(G)$ is the Hopf-von Neumann algebra generated by the left regular representation of G. To a $\mathscr{L}(G)$-comodule we shall associate a dual covariance algebra $\mathfrak{A}$ and a natural covariant system $(\mathfrak{A}, \varrho, G)$, and in Theorem 1 the covariant systems coming from $\mathscr{L}(G)$-comodules are characterized. In [2] it was shown that the covariance algebra of a covariant system in a natural way is a $\mathscr{L}(G)$-comodule. Therefore one can form the dual covariance algebra of a covariance algebra and the covariance algebra of a dual covariance algebra. Theorems 2 and 3 deal with these algebras – generalizing a result by Takesaki. As an application we give a new proof of a theorem by Digernes stating that the commutant of a covariance algebra itself is a covariance algebra and prove the similar result for dual covariance algebras.

§ 1. Introduction

If G is a locally compact group and $\varrho: G \to \mathrm{Aut}(A)$ is a continuous homomorphism of G into the group of *-automorphisms of a von Neumann algebra A, (A, ϱ, G) is called a covariant system and one can form the covariance algebra $\mathfrak{A} = W^*(A, \varrho, G)$. Takesaki showed in [8] that if G is abelian there is a natural covariant system $(\mathfrak{A}, \tau, G^\wedge)$ over the dual group $G^\wedge$ and that $W^*(\mathfrak{A}, \tau, G^\wedge) \cong A \otimes \mathscr{B}(L^2(G))$, i.e. the tensorproduct of A with the algebra of all bounded operators on $L^2(G)$.

For a non-abelian G there is no dual group to act on the covariance algebra $\mathfrak{A} = W^*(A, \varrho, G)$, but this author showed in [2] that the natural structure on $\mathfrak{A}$ corresponding to the action of a dual group is that of a $\mathscr{L}(G)$-comodule. There one used that $\mathscr{L}(G)$, the von Neumann algebra generated by the left regular representation of G is a Hopf-von Neumann algebra, cf. [7].

So if A is a von Neumann algebra, what seems to correspond to a covariant system on A over $G^\wedge$ if G is abelian is that of a $\mathscr{L}(G)$-comodule structure on A.