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KMS Conditions and Local Thermodynamical Stability of Quantum Lattice Systems

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Abstract. We formulate local thermodynamical stability conditions for states of quantum lattice systems, and show that these conditions are implied by, and in the case of translationally invariant states equivalent to, those of Kubo-Martin-Schwinger (KMS).

1. Introduction

This paper is concerned with the relationship between certain local thermodynamic stability (LTS) conditions and the KMS conditions for quantum lattice systems.

The LTS conditions, which will be precisely specified below, may be described as follows. For each state ϕ of a system and each bounded region Λ, we define a conditional free energy $\tilde{F}_\Lambda(\phi)$ (cf. Definition 2.1): this quantity is a quantal generalisation of that defined in [1, 2] for classical systems, and is designed to represent the free energy of the "open system" consisting of the particles in Λ, interacting with one another and with the particles outside that region. We define the LTS conditions for ϕ (cf. Definition 2.2) to be that, for each bounded region Λ, $\tilde{F}_\Lambda(\phi)$ is minimal for variations in the state which leaves it unchanged outside Λ.

With these definitions, and under the assumption of tempered, translationally invariant, finite-body (or somewhat more general) interactions, we prove the following Theorem.

Theorem. (a) *If a translationally invariant state satisfies the LTS conditions, it satisfies the KMS conditions.*

(b) *A state satisfying the LTS conditions is stationary in time.*

(c) *A state satisfies the LTS conditions if it satisfies the KMS conditions.*

We shall adopt the notations of references [3, 4]. Thus, $\mathfrak{A}$ denotes the C^*-algebra of quasi-local observables of the system and $\mathfrak{A}(I)$ its subalgebra for the

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